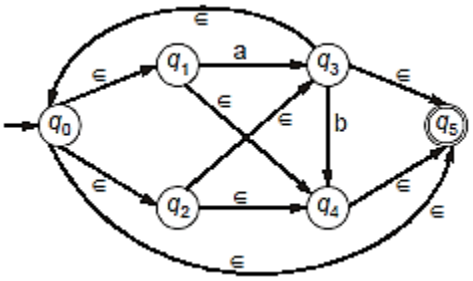
THEORY OF COMPUTATION

SOLUTIONS

1. Consider the following NFA.



Find the epsilon closure of state q_2.

(a) $\{q_4, q_3, q_5\}$ only
(b) $\{q_3, q_4\}$ only
(c) $\{q_0, q_5\}$ only
(d) $\{q_0, q_1, q_2, q_3, q_4, q_5\}$

Solution: Option (d)

Explanation:

(d)

Null closure of any state are those state which are reachable only by ∈-transition, i.e., here q_2 to q_3 and q_4 by ∈-transition, from q_3 we can reach q_0 and q_5, and from q_0 to q_1 by using ∈-transition.

So ∈-closure of q_2 is $\{q_0, q_1, q_2, q_3, q_4, q_5\}$.

2. Let L be a language defined as L = {w | w contain 'aba' as substring, w ∈ (a + b)*}. Which of the following DFA will recognize the language L?

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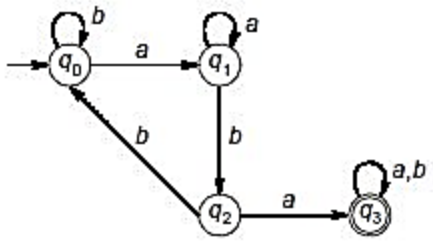
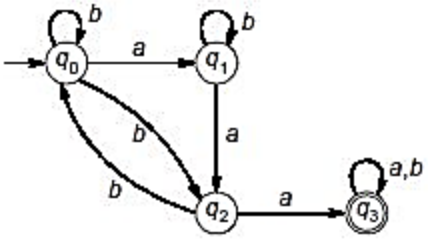
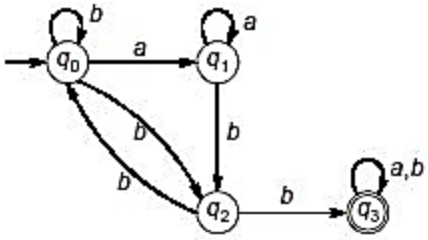
(a)



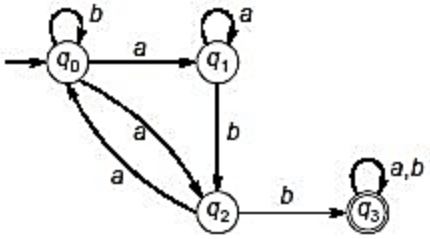
(c)



(b)



(d)

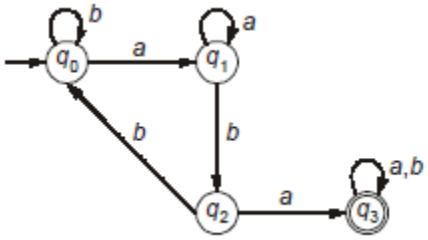


Solution: Option (a)

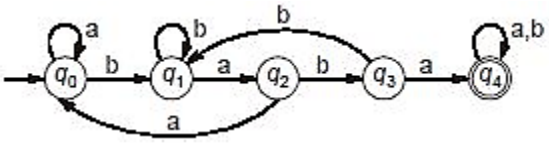
Explanation:

(a)

The correct DFA is



3. Consider the following DFA.



Which of the following will represent the complement of the language generated by above DFA?

(a) L = {w | w contain abab}
(b) L = {w | w contain baba}
(c) L = {w | w does not contain baba}
(d) L = {w | w does not contain abab}

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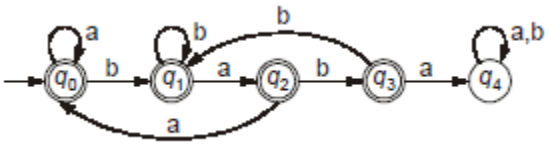
Solution: Option (c)

Explanation:

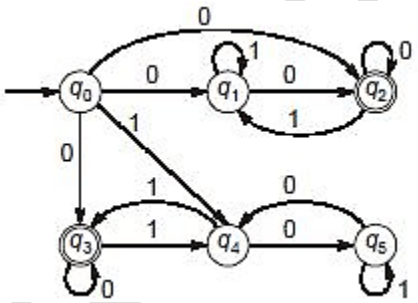
(c)

Complement of above DFA is generate after changing the final state to non-final state and non-final state to final state, which is given below



This DFA is representing the strings which do not contain "baba" as substring.

4. Consider the following NFA.



The above NFA accepts all those binary strings which represents the decimal numbers and are

(a) divisible by 6 only
(b) divisible by 2 and 3 only
(c) divisible by 2 or 3
(d) None of these

Solution: Option (c)

Explanation:

(c)

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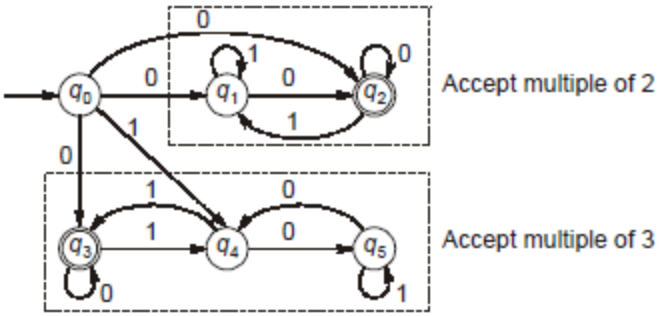

NFA produce strings like 0, 010, 0110, 11 etc. which are divisible by 2 or 3.

5. Which of the following statements are True?

S1: Every left recursive grammar can be converted to right recursive grammar and vice-versa.
S2: All ∈-productions can be removed from any context free grammar.
S3: An unambiguous context free grammar always has a unique parse tree for each string of the language generated by it.

(a) Only S1 and S2
(b) Only S1 and S3
(c) Only S2 and S3
(d) S1, S2 and S3

Solution: Option (b)

Explanation:

Statement S1 and S3 are True.

S2 is false because all ∈-production can be removed from grammar only when the languages do not contain ∈-string but if language contain ∈-string then removal of the null productions is not possible.

6. Which one of the following regular expression over {0, 1} denotes the set of strings which do not contain 101 as a substring?

(a) 0^*100^+
(b) $1^*(01+0)^*$
(c) 10^*10
(d) $0^*(1 + (00)^+)^*0^*$

Solution: Option (d)

Explanation:

(d)

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(a) Regular expression = 0^*100^+ can generate strings that do not contain 101 as substring but it cannot generate $\in$, 0, 1, 01, 10, 110, ... So it is not the right answer.
(b) Regular expression = 1*(01 + 0)* contain 101 as string which can contain 101 as substring.
(c) Regular expression = 10*10 contain 1010 as string which can contain 101 as substring.
(d) Regular expression = $0^*(1 + (00)^+)^*0^*$ can generate all strings which does not contain 101 as substring.

7. If string of length 10 is used to test for membership, then the number of table entries in CYK algorithm is

(a) 50
(b) 55
(c) 45
(d) 99

Solution: Option (b)

Explanation:

(b)

By theorem if n is the length of string to test for membership, then the number of table entries in CYK algorithm is n(n+1)/2

So, $10 \times \frac{11}{2} = 55$

8. Let <M> be the encoding of Turing machine as a string over $\Sigma = \{0, 1\}$. Let L = {<M> | M is TM on input w will visit some state P}.

The language L is

(a) Decidable
(b) Undecidable but partially decidable
(c) Undecidable and not even partially decidable
(d) Not a decision problem

Solution: Option (b)

Explanation:

(b)

Language L is state entry problem. Halting problem of Turing machine can be reduced to state entry problem. Since halting problem of Turing machine is undecidable the state entry problem

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must also be undecidable. The language corresponding to state entry problem is RE but not REC. So L is undecidable but partially decidable.

9. Let $L \leq_M L'$ denote the language L is mapping reducible (many to one reducible) to language L'. Which one of the following is True?

(a) If $L \leq_p L'$ and L' is semidecidable then L is semidecidable.
(b) If $L \leq_p L'$ and L is RE then L' is RE.
(c) If $L \leq_p L'$ and L is decidable then L' decidable.
(d) If $L \leq_p L'$ and L is recursive.

Solution: Option (a)

Explanation:

(a)

$L \leq pL'$. Since L' is semidecidable then L is semidecidable is one way theorem (semidecidability goes backward).

10. Let L be the language containing only the string S where

$$S = \begin{cases} 0 & \text{If you will never clear the gate} \\ 1 & \text{If you will clear the gate some day} \end{cases}$$

Which of the following is true? (L' is the Complement of language L)

(a) L is decidable
(b) L' is decidable
(c) L and L' both are decidable
(d) L is undecidable

Solution: Option (c)

Explanation:

(c)

The language L is one of the two languages {0}, {1}. In either case the language is finite only. Hence L is decidable. Since complement of decidable is decidable only.

So L and L' both are decidable.

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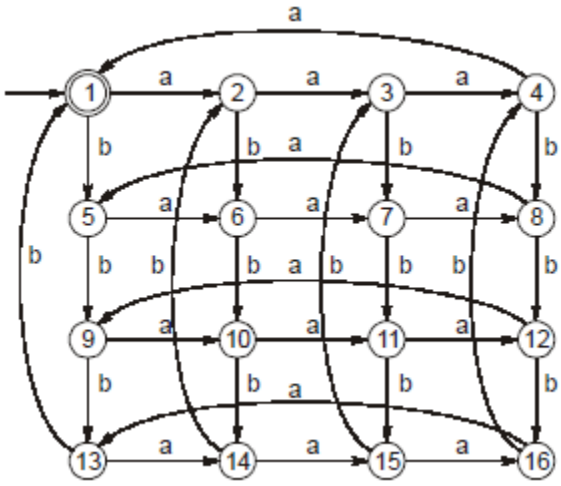
11. In the minimal finite automata accepting the set of all strings over {a, b} where the number of a's and the number of b's is divisible by 4 the total number of non-final states are _____.

Solution: 15

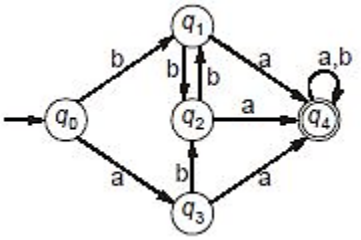
Explanation:

(15)



Total number of states = m × n = 4 × 4 = 16, where, m = Number of a's and n = Number of b's. Only state 1 is final and other states from 2 to 16 are non-final.

12. Consider the following DFA D.



The number of states in the minimization of D is _____.

Solution: 3

Explanation:

(3)

The minimized DFA after combining the q_1, q_2 and q_3 are given below.

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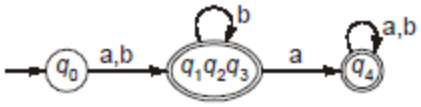
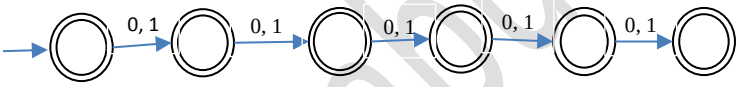


13. Let w be any string of length 5 in (0 + 1)*. Let L be the set of all substrings of w. The minimum number of states in non-deterministic finite automaton that accept L is _____.

Solution: 6

Explanation:

Given w be any string of length 5 in (0+1)*. So, w can be any of the strings 00000, 00001, 00010, 00011, …
L is a set of all substrings of w
L = {∈, 0, 1, 00, 10, 01, 11, 000, 001, 010, 011, ... , 11111}



Therefore, there are 6 states in the NFA and all are final states.

14. If G is a context free grammar and w is a string of length 10 in L(G). The length of derivation of w in G, if G is in Chomsky normal form is _____.

Solution: 19

Explanation:

(19)

A context free grammar G is in Chomsky normal form if all productions are in one of two simple form, either:

1. A → BC where A, B and C are variables, or
2. A → a where A is a variable and a is a terminal

So for any string of length n first production of type A → BC is used n – 1 times to produce sentential form of length n containing only variables and then each variable is replaced by a terminal using productions of

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type A → a, n times. So the length of derivation of string w of length n in CNF is

(n – 1) + n = 2n – 1
2n – 1 = 2 × 10 – 1
= 20 – 1 = 19 steps

15. Number of states in the minimized DFA that accepts all strings over alphabet Σ = {0, 1} in which number of 0's is divisible by 8 or number of 0's is divisible by 16 is _____.

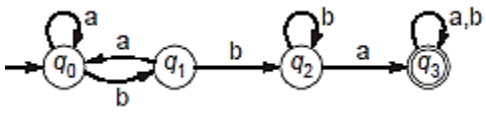
Solution: 8

Explanation:

(8)

Number of states = HCF (8, 16) = 8

16. Consider the following DFA.



The number of strings upto length 5 where first and last character of string is 'a' are _____.

Solution: 4

Explanation:

(4)

Strings of one length = 0 → Not possible
Strings of two length = 0 → Not possible
Strings of three length = 0
Strings of four length = abba … (1 string)
Strings of five length = abbba, abbaa, aabba … (3 strings)

Total 4 strings possible.

17. Consider the following FA

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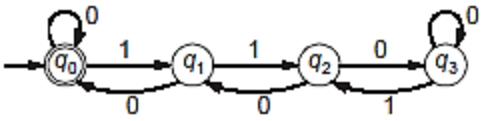


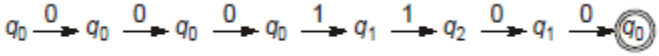
Which of the following string is accepted by above FA?

(a) 0110
(b) 00110
(c) 000111
(d) 0001100

Solution: Option (d)

Explanation:

(d)



String 0001100 accepted by FA.

18. Which of the following is True?

(a) Complement of CSL is CSL
(b) Complement of CFL is CFL
(c) Complement of RE is RE
(d) Complement of non-CFL can't be CFL

Solution: Option (a)

Explanation:

(a)

CSL closed under complement so complement of CSL is CSL.
RE and CFL are not closed under complement.
Complement of non-CFL can be CFL i.e. {ww = CSL} complement = CFL.

19. Consider the following statements:

S1: Infinite language with finite dependency is always regular.
S2: Regular expression 01*0 represents an infinite set of finite strings.

Which of the following is True about S1 and S2?

(a) only S1 is True
(b) only S2 is True

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(c) both S1 and S2 are True (d) both S1 and S2 False

Solution: Option (c)

Explanation:

(c)

Statement S1 is true.
Example: $\{a^* b^n c^n d^* \mid n \leq 2\}$, here comparison between 'b' and 'c' is finite but the language is infinite. So it is regular language.

The statement S2 is true.
Given regular expression is infinite set (because of *) of finite strings. A regular expression can generate any infinite length string (since string length always be finite but language can be infinite).

20. Consider the following

S1: Pumping lemma is used to prove, that particular language is not regular
S2: For all DCFL there exist LR(k) grammar but LL(k) may not exist.

Which of the above statements are true?

(a) Only S1 (b) Only S2
(c) Only S1 and S2 (d) None of these

Solution: Option (c)

Explanation:

(c)

S1: Pumping lemma is always used in negative sense to prove language is not regular.
S2: This is a relationship between DCFL and LR(k).

DCFL ↔ LR(k)
but for DCFL and LL(k)

LL(k) → DCFL
S2 also correct.

21. Consider the following statements:

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1. The complement of every Turing recognizable language is Turing recognizable.
2. Deciding if a given string is generated by a given context free grammar is decidable.

Which of the above statements are correct?

(a) Only 1
(b) Both 1 and 2
(c) Only 2
(d) Neither 1 nor 2

Solution: Option (c)

Explanation:

(c)

1. Turing recognizable language are RE language which are not closed under complementation. So statement is false.
2. Second statement is true.

22. Consider the following language L = {w ∈ (a+b)* | w has atleast as many occurrences of (bba)'s as (abb)'s}. Which of the following statements is/are true?

S1: Language L is regular.
S2: Complement of L is CFL.
S3: Complement of L is CSL.
S4: Reversal of L is CFL.

(a) Only S1
(b) Only S2 and S4
(c) Only S1 and S3
(d) All of these

Solution: Option (d)

Explanation:

Machines which accept L needs only finite amount of memory. Since there is no need to keep the number of bba's in the memory because whenever two abb's comes together (adjacent), then one bba's always come between them. So language L is regular. Since regular language is closed under complement, reversal and regular language are subset of CSL and CFL.

So, all the statements are correct.

23. Consider the following grammar G.

G: $S \rightarrow XC \mid AY$

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$X \rightarrow aXb \mid \in$
$C \rightarrow cC \mid c$
$A \rightarrow aA \mid a$
$Y \rightarrow bYc \mid \in$

Which of the following is correct?

(a) G is ambiguous but L(G) is not inherently ambiguous
(b) G is ambiguous but L(G) is inherently ambiguous
(c) G is not ambiguous but L(G) is not inherently ambiguous
(d) None of the above

Solution: Option (b)

Explanation:

(b)

The grammar generates the following language.

$$L(G) = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^n b^m c^m \mid n, m \geq 0\}$$

Which is a standard example of inherently ambiguous language i.e. no grammar is possible which is unambiguous. Any string in this language is of the form of anbncn and will have 2 derivation trees. Whatever be the grammar, one of these derivation trees will start with $S \rightarrow XC$ and another will start with $S \rightarrow AY$.

24. Consider the language

$$L_1 = \{0^n 1^n 2^m / \ n, m \geq 0\} \text{ and } L_2 = \{0^n 1^m 2^m / \ n, m > 0\}$$

Which of the following statements is True? (L^c is the compliment of L)

(a) $L_1 \cap L_2$ is CSL
(b) $L_1 \cap L_2$ is CFL
(c) $L_1^c \ L_2^c$ is CFL
(d) None of these

Solution: Option (a)

Explanation:

(a)

The intersection of L_1 and L_2 is given by $L_1 \cap L_2 = \{0^n 1^n 2^n \mid n \geq 0\}$ which is well known CSL.

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25. Consider the following context free grammar G:

G: $R \rightarrow XRX \mid S$
$S \rightarrow aAb \mid bAa$
$A \rightarrow XAX \mid X \mid \in$
$X \rightarrow a \mid b$

Which of the following correctly describes the above grammar G?

(a) L(G) contain all strings over ‘a’ and ‘b’ that are palindrome only
(b) L(G) contain all strings over ‘a’ and ‘b’ that are not palindrome
(c) L(G) contain all strings with equal number of ‘a’ and ‘b’ only
(d) None of the above

Solution: Option (b)

Explanation:

(b)

Assume that there are two palindrome strings one of them is even palindrome and one is odd palindrome i.e. abba and ababa respectively. The grammar will not generate both the strings. The even palindrome contain equal number of ‘a’ and ‘b’ here. So option ‘a’ and ‘c’ are not correct. The grammar will generate all strings over ‘a’ and ‘b’ that are not palindrome. Therefore (b) is true since it will generate all strings which are not palindrome.

26. Consider the following statements:

S1: Turing machine can write the blank symbol □ on its tape
S2: Tape alphabet ⌈ can be same as input alphabet Σ
S3: Turing machine’s head can ever be in the same location in two successive steps
S4: Turing machine contain only single state

Which of the above statements are correct?

(a) S1 and S2 only
(b) S1 and S3 only
(c) S2 and S4 only
(d) S1 and S4 only

Solution: Option (b)

Explanation:

(b)

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S1: The tape alphabet can contain blank □. A Turing machine can write any character in tape alphabet on its tape.

S2: Input alphabet Σ never contains blank symbol □ but tape alphabet ⌈ contain.
So, Σ ≠ ⌈ incorrect.

S3: If Turing machine attempts to move its head off the left hand end of the tape, it remains on the same tape cell. So, correct.

S4: Any Turing machine must contain two different state q_{accept} and q_{reject}. So a Turing machine contains atleast two state. So, incorrect.

Only S1 and S3 are correct.

27. Find the minimum number of states in the DFA which accept the language of all strings that begin or end with 00 or 11.

(a) 6 (b) 7
(c) 8 (d) 9

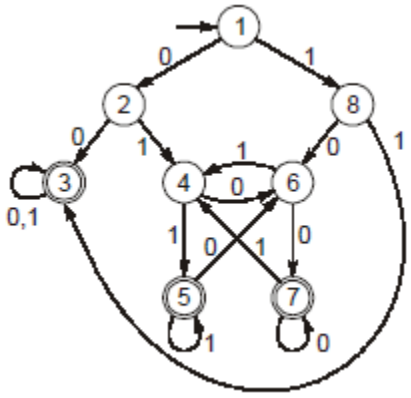
Solution: Option (c)

Explanation:

(c)

This language contains all the strings that either begin or end with 00 or with 11. So the set of strings that will be accepted by DFA are 11010, 001, 1011, 10100, …

The final state will be the state in which we land when the beginning of strings either 00 or 11 and states in which we land when the end of DFA is either 00 or 11. DFA of the language is given below.



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28. How many states are present in minimal DFA that accept all the strings of the regular expression b*a(a+b)*?

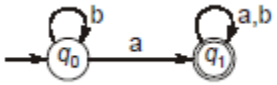
(a) 2 (b) 1
(c) 4 (d) 5

Solution: Option (a)

Explanation:

(a)

The minimal DFA for regular expression b*a(a+b)* is



29. Let L be the language represented by the regular expression Σ*abbΣ* where Σ = (a + b). What is the minimum number of states in FA that recognize L?

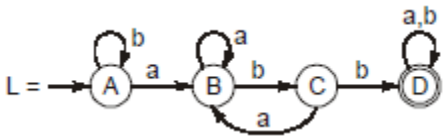
Solution: 4

Explanation:

(4)

Σ*abbΣ* = (a+b)*abb (a+b)*

Every string of the language L contains abb as substring. To accept the minimal string "abb" we need 4 states since substring machine don't required trap state we need 4 state only.



30. Let L_1 = a*b* and L_2 = {ab}. L_3 = Prefix (L_1* ∩ L_2), where prefix (L) = {uluv ∈ L for any v}. The number of strings in L_3 is _____.

Solution: 3

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Explanation:

(3)

$L_1 = a^*b^* \Rightarrow L_1^* = (a^* b^*)^* = (a + b)^*$
$L_2 = \{ab\}$
$L_1^* \cap L_2 = (a + b)^* \cap \{ab\} = \{ab\}$
$L_3 = \text{Prefix}(L_1^* \cap L_2) = \{\in, a, ab\}$

31. Consider the following regular expression R = a*b* + b*a*. The number of equivalence classes of Σ* to represent a language which is equivalent to R is _____.

Solution: 6

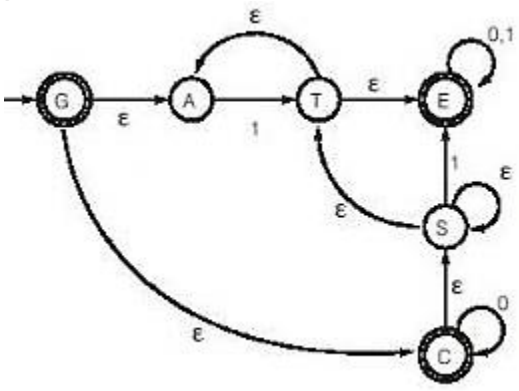
Explanation:

(6)

[1] = {ε}
[2] = {aa*}
[3] = {aa* b b*}
[4] = {bb*}
[5] = {bb* aa*}
[6] = {(aa*bb*a + bb*aa*b) (a + b)*}

∴ 6 equivalence classes = 6 states in minimized DFA

32. Consider the following ∈-NFA.



The number of states required to construct an equivalent minimized DFA for the given ∈-NFA is _____.

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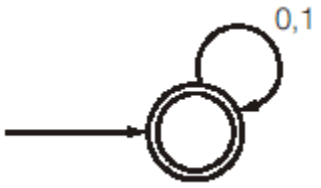
Solution: 1

Explanation:

(1)

From G to E can be reachable without reading any input and then can read any sequence of 0's and 1's = (0 + 1)*

∴ one state is required for DFA



33. Consider the following CFG.

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \in$$

For the above CFG, the total number of strings generated whose length is less than or equal to 8 [exclude the empty string] is _____.

Solution: 60

Explanation:

(60)

$n = 1 \Rightarrow L(G) = \{a, b\} \Rightarrow \#\text{ strings} = 2$
$n = 2 \Rightarrow L(G) = \{aa, bb\} \Rightarrow \#\text{ strings} = 2$
$n = 3 \Rightarrow L(G) = \{aaa, aba, bab, bbb\} \Rightarrow \#\text{ strings} = 4$
$n = 4 \Rightarrow L(G) = \{aaaa, abba, baab, bbbb\} \Rightarrow \#\text{ strings} = 4$

Similarly,
$n = 5 \Rightarrow \#\text{ strings} = 8$
$n = 6 \Rightarrow \#\text{ strings} = 8$
$n = 7 \Rightarrow \#\text{ strings} = 16$
$n = 8 \Rightarrow \#\text{ strings} = 16$

Total # strings = 2 × [2+4+8+16] = 2 × [30] = 60 strings

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